Subatomic Physics 2006/2007

Exam
Friday, 09 - February - 2007, 14:00 - 17:00

70 points
(Please mark name and student number on every sheet.)

1 Main Exam

1.1 Our Sun (15 points)

At present the sun has a mass of $M_S \approx 2 \cdot 10^{30}$ kg (which is about $3.3 \cdot 10^5$ earth masses M_E). The solar surface is composed of 71% hydrogen (H), 27% helium (^{4}He), and 2% heavier elements (percentage with respect to mass). The sun radiates energy at a total power $P_S = 4 * 10^{26} W$. The main energy producing mechanism in the center of the sun is the so called *proton-proton cycle*:

(i)	$p + p \rightarrow d + e^+ + \nu_e$	$+E_1$;	$E_1 = +0.42 MeV$
(ii)	$p + d \rightarrow ^3\text{He} + \gamma$	$+E_2$;	$E_2 = +5.49 MeV$
(iii)	$^3\text{He} + ^3\text{He} \rightarrow p + p + \alpha$	$+E_3$	
(iv)	$e^+ + e^- \rightarrow 2\gamma$	$+E_4$;	$E_4 = +1.02 MeV$
(v)	The sum of one full cycle is hence:		
	$4p \rightarrow \alpha + 2e^+ + 2\nu_e + 2\gamma$	$+E_5$	

Let's assume that the sun was formed in a very short period of time compared to its age and that the atoms it may have collected from the universe due to gravitation since is negligible. Let's assume further it runs constantly since its beginning for all time since.

(a) Which interactions are responsible for every of the steps (i) to (iv) in the *proton-proton cycle*?

(b) Please draw the Feynman diagrams for the processes (i) to (iv) at the quark level.

(c) How much energy is released in step (iii) (calculate E_3.), and in one full cycle (calculate E_5)?

(d) How much hydrogen is converted into helium every second?

(e) What percentage of hydrogen has been used up since the sun started $5 \cdot 10^9$ years ago?

(f) Do you have an explanation for the percentage of H and He found at the surface presently?

(g) How much mass did the sun loose since its birth $5 \cdot 10^9$ years ago? Compare this with the mass of the earth.

1.2 Neutrino Experiments (15 points)

(a) The distance sun-earth is $1.5 \cdot 10^5$ km. What is the flux Φ_ν of neutrinos at the earth which originate from the process $p + p \rightarrow d + e^+ + \nu_e$?

(b) The GALLEX experiment measures solar neutrinos through the reaction $^{71}_{3}1Ga + \nu_e \rightarrow^{71}_{3} 2Ge + e^-$. The cross section for the reaction is $2.5 \cdot 10^{-21} b$. One looks for radioactive ^{71}Ge atoms ($T_{1/2} = 11.2$ days) in a tank with 30 tons of dissolved chemically highly pure $GaCl_4$ (Ga:40% ^{71}Ga and 60% ^{69}Ga, Cl: 50% ^{35}Cl and 76%^{37}Cl). 24% of the neutrinos have an energy above threshold. How many ^{71}Ge atoms are produced per day?

(c) The Ga atoms can be removed from the tank in a flushing cycle which is very short compared to $T_{1/2}$ and collected in a counter for radioactivity. What is the expected ^{71}Ge (radio) activity that can be collected (i) after a running time of the experiment of three days ($T_3 = 3d$) and (ii) after a running time T_l which is very long compared to the lifetime of the isotope?

1.3 Weak Interactions (10 points)

The weak eigenstates and the mass/flavour eigenstates of quarks are not identical.

(a) How are they related to each other? Explain briefly.

(b) What is the situation for leptons? Explain briefly.

(c) Give one example each of a (i) leptonic, (ii) semileptonic and (iii) non-leptonic weak interaction process. Draw the corresponding Feynman diagrams and explain how the coupling constants differ, which describe the vertices.

(d) The so called *neutral currents* were important to manifest the Standard Model. These *currents* are a name for the exchange of a Z^0-boson. Why could they be unambiguously observed in the process (i) $\nu_\mu + e^- \rightarrow \nu_\mu + e^-$ and not in the process (i) $\nu_e + e^- \rightarrow \nu_e + e^-$? (*Hint: Think about all possible weak processes with the given initial and final states.*)

1.4 Range of Forces (5 points)

Forces are mediated by the exchange of particles. (i) Estimate the range of the weak interactions and of the nuclear force mediated by charged pions. (ii) Indicate your line of reasoning. (iii) Why are forces mediated by bosons?

1.5 Allowed and Forbidden Processes (10 points)

Which of the following processes are allowed and which are forbidden. Please give reasons for your judgment. For allowed processes give the interaction

(a) $p + \overline{p} \rightarrow \pi^+ + \pi^- + \pi^0 + pi^- + pi^+$

(b) $\overline{\nu_\mu} + p \rightarrow \mu^+ + n$

(c) $\nu_e + p \rightarrow e^+ + K^0 + \Lambda^0$

(d) $\mu^+ + e^- \rightarrow \mu^- + e^+$

(e) $\mu^- \rightarrow e^- + \nu_\mu + \nu_e$

(f) $p \rightarrow \pi^+ + e^- + e^+ + \gamma$

(g) $\mu^- \rightarrow e^- + e^+ + e^- \nu_\mu + \overline{\nu_e}$

(h) $J/\Psi(2^1S_0) \rightarrow J/\Psi(1^1S_0) + 3\gamma$ (remember: $J/\Psi = (c\overline{c})$ bound state)

(i) $p + \overline{p} \rightarrow c\overline{c}$

(h) $p +^{12} C \rightarrow^{12} C + n + \pi^+ + \pi^- + \pi^+$

1.6 Radioactivity - Alpha decay (15 points)

The binding energy of an α-particle is 28.3 MeV. In the framework of the liquid drop model you can estimate from which mass number A onward α-decay is possible for all nuclei. What is this value? You may neglect terms such as atomic binding and pairing energies (please explain why that makes sense).

2 Constants, Relations and Conversion Factors

2.0.1 Constants of Relevance

Speed of light	c	2.998	m/s
Planck constant	h	$4.136 \cdot 10^{-24}$	GeVs
	$\hbar = \frac{h}{2 \cdot \pi}$	$6.582 \cdot 10^{-25}$	GeV/c
Electron charge	e	$1.602 \cdot 10{-}19$	C
Electron mass	m_e	0.510998918(44)	MeV/c^2
Proton mass	m_p	938.272029(80)	MeV/c^2
Neutron mass	m_n	939.565360(81)	MeV/c^2
Deuteron mass	m_d	1875.61282(16)	MeV/c^2
Helium-3 mass	$m_{^3He}$	2809.41334(24)	MeV/c^2
Alpha particle mass	m_α	3727.37917(32)	MeV/c^2
Electron neutrino mass	m_{ν_e}	< 2.2	eV/c^2
Muon mass	m_μ	105.658369(9)	MeV/c^2
Charged Pion mass	$m_{\pi^\pm}$	139.57018(35)	MeV/c^2
W$^\pm$- boson mass	m_W	80.403(29)	GeV/c^2
Z^0-boson mass	m_Z	91.1876(21)	GeV/c^2

2.0.2 Relations

Mass of Atom (Bethe-Weizaecker):

$$M(A,Z) = Nm_n + Zm_p + Zm_e - a_v A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + a_a \frac{(N-Z)^2}{4A} + \frac{\delta}{A^{1/2}}$$

with

$a_v = 15.67 MeV/c^2$

$a_s = 17.23 MeV/c^2$

$a_c = 0.714 MeV/c^2$

$a_a = 93.15 MeV/c^2$

$\delta = 0$ (odd A) or -11.2 MeV/c^2 (Z and N even) or +11.2 MeV/c^2 (Z and N odd)

2.0.3 Conversion Factors

Electronvolt	eV	$1.60217653(14) \cdot 10^{-19}$	J
Tesla	T	$0.561 \cdot 10^{30}$	MeV/(c$^2 \cdot$ C $\cdot$ s)
Kilogram	kg	$5.60958896(48) \cdot 10^{35}$	eV/c^2
barn	b	$1 \cdot 10^{-28}$	m^2

Note: For some of the questions different approaches are possible, such that you may not necessarily need all of the given constants and equations. Unless differently stated, the final results are sufficient, if given to 3 significant figures (3 leading digits).